

# Gas and Liquid Phase Sensitivity of “Love” Waves

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## Acknowledgements

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# Overview

## 1. Layer-Guided Acoustic Waves

- Love waves v SH-APMs
- Dispersion curves and phase speed mass sensitivity
- Dispersion and group velocity
- Polymer waveguides and the “Sauerbrey” equation

## 2. Experimental Data

- Existence of layer-guided SH-APMs
- Group velocity mass and liquid sensitivity

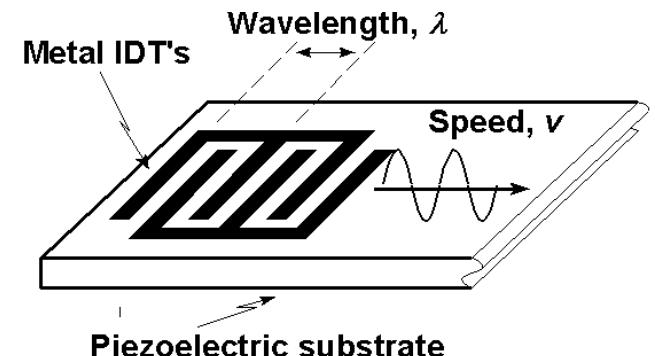
## 3. Love Waves and Frequency

- Higher frequency and multiple modes
- Frequency hopping

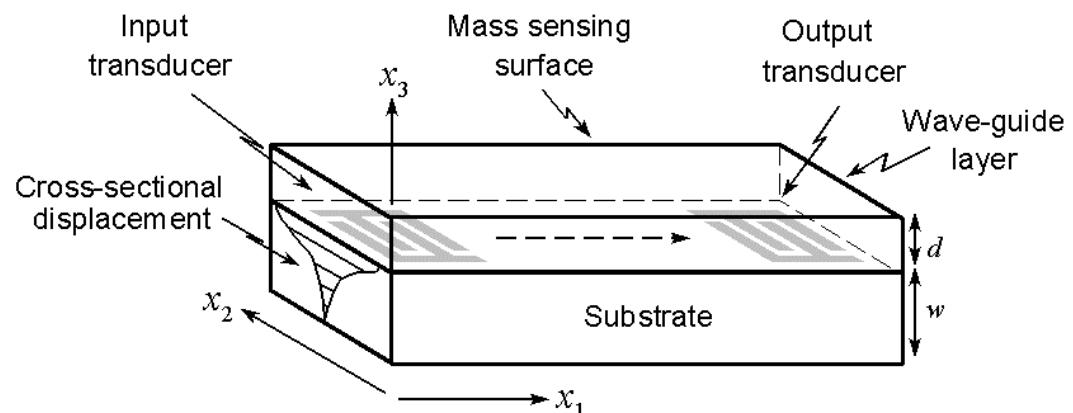
## 4. Summary

# Love Waves v SH-APMs

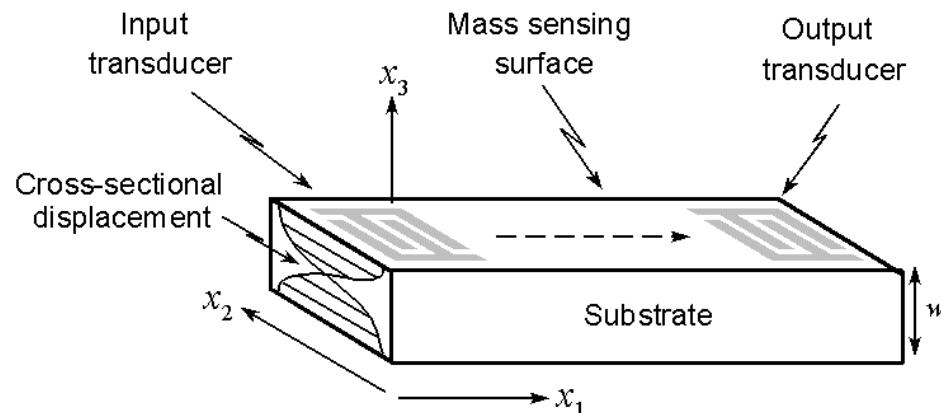
- Surface Acoustic Wave (SAW)



- Love Wave  
Layer guided SH-SAW  
with  $v_l < v_s$



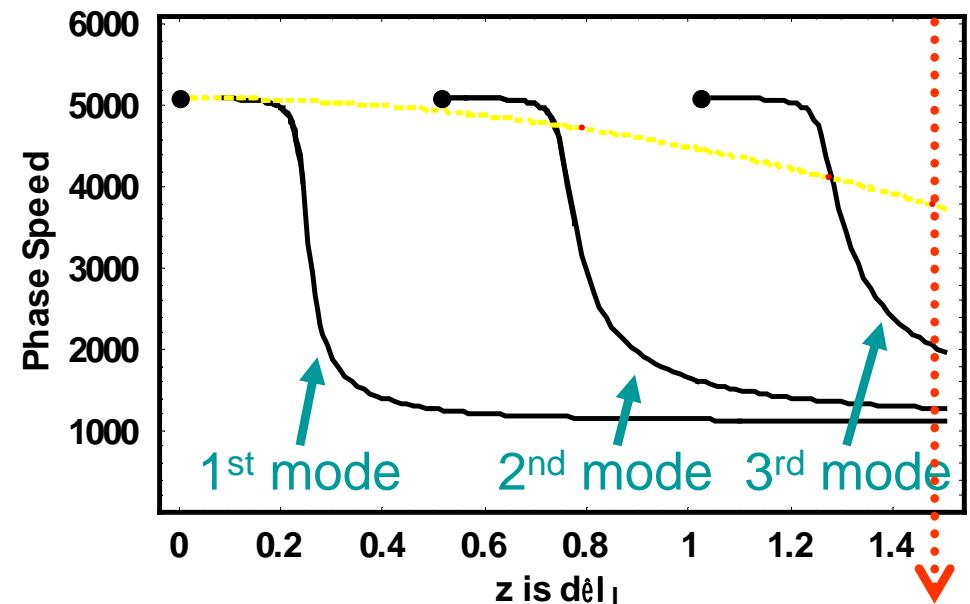
- SH-APM  
Substrate resonance



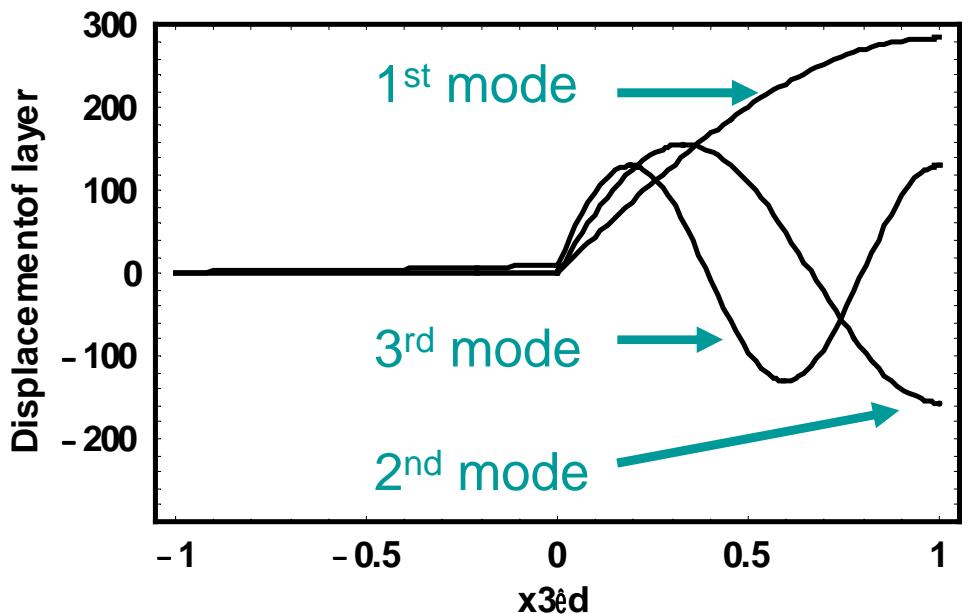
# Love Waves

- Theoretical dispersion curve

(Insertion loss is unchanged by an elastic guiding layer)



- Displacements for first three modes ( $z=1.3$ )



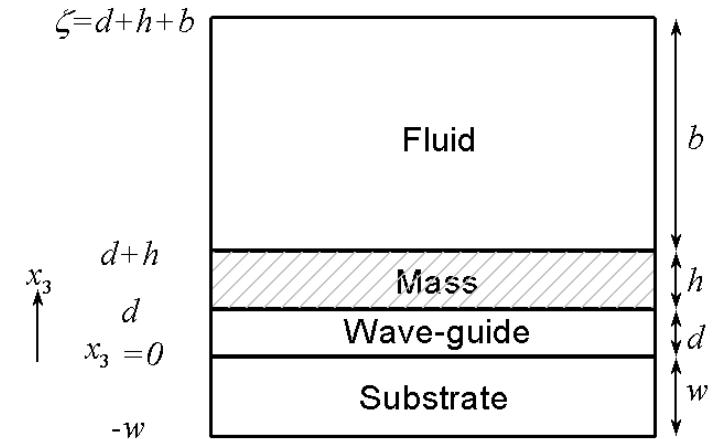
# Layer-Guided SH-APMs

- Generalized Dispersion Equation<sup>1</sup>  
Layer and substrate displacements

$$\underline{u}_l = (0, 1, 0) [A e^{-jT_l x_3} + B e^{jT_l x_3}] e^{j(\omega t - k_l x_1)}$$

$$\underline{u}_s = (0, 1, 0) [C e^{T_s x_3} + D e^{-T_s x_3}] e^{j(\omega t - k_s x_1)}$$

Eqns of motion  $\Rightarrow T_l$ 's and  $T_s$



Boundary conditions  $\Rightarrow$  dispersion eqn

- Substrate + Layer Solutions

$T_s$  real  $\Rightarrow \nu < \nu_s \Rightarrow$  "Love" Waves

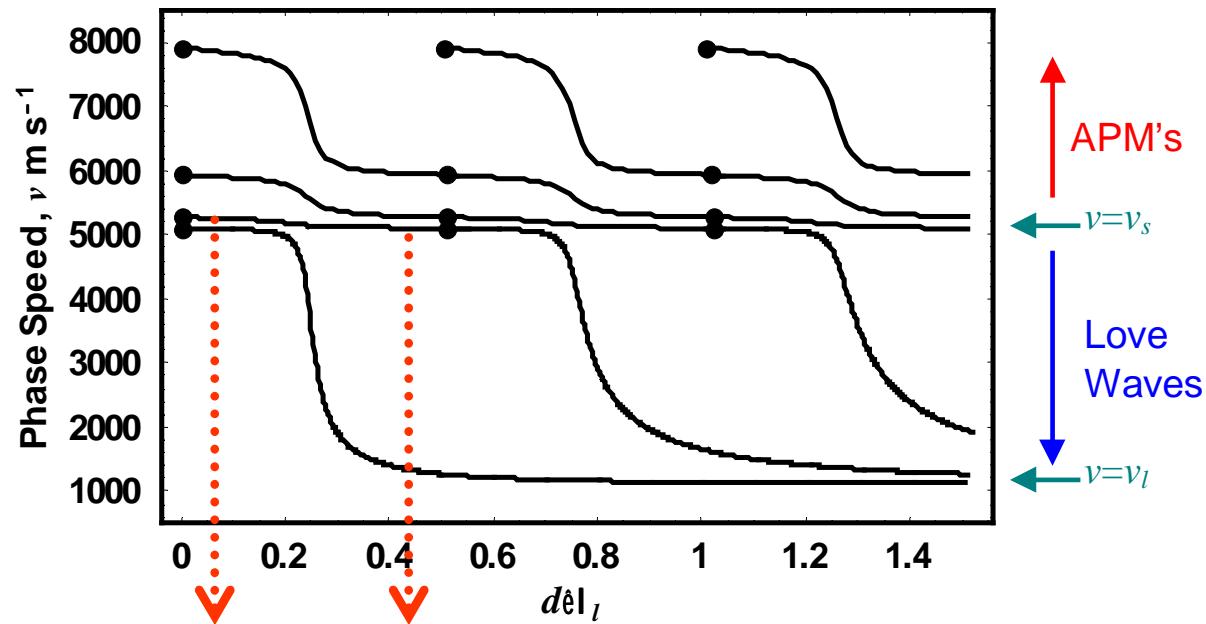
$T_s = jk_s$  with  $k_s$  real  $\Rightarrow \nu > \nu_s \Rightarrow$  "Layer guided SH-APMs"

<sup>1</sup>McHale et al, Europhys. Lett. (2002) 58, 818-822, J. Appl. Phys. (2002) 91, 5735-5744.

McHale et al, "Mass, liquid and polymer sensitivity .....", Accepted J. Appl. Phys. (2002).

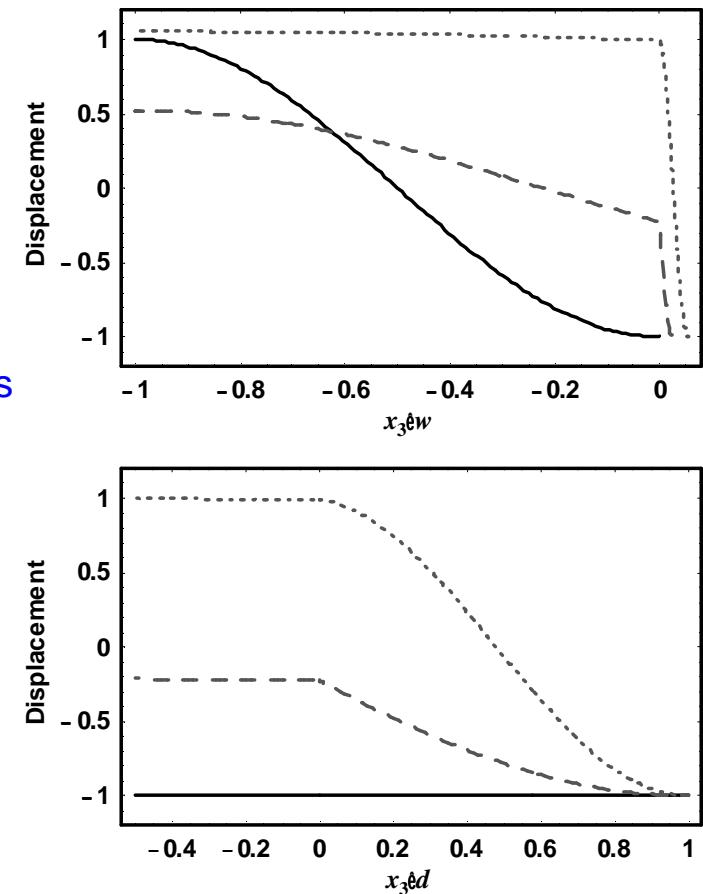
# Substrate + Mass Guiding Layer

Dispersion Curve



Points = Anti-node moving from substrate to layer

Evolution of 1st SH-APM



Solid  $\rightarrow$  dashed  
with increasing guiding

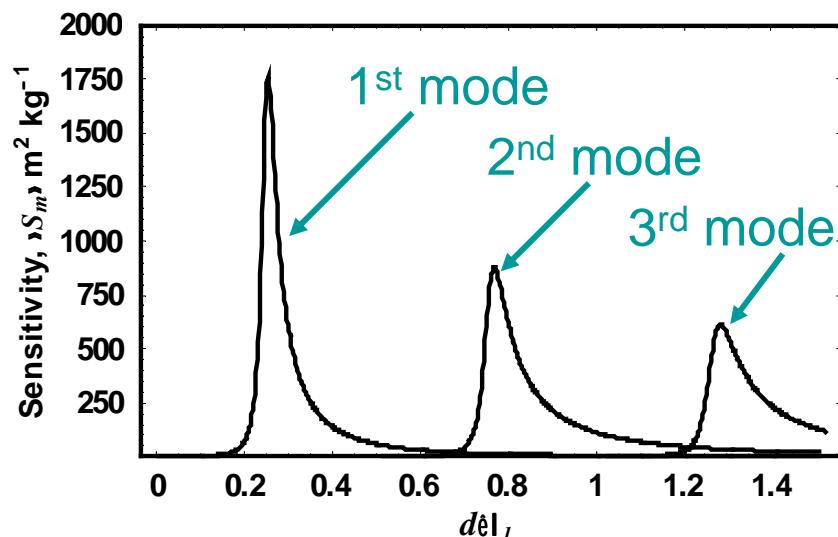
# Phase Speed Mass Sensitivity

$$S_m = \lim_{\Delta m \rightarrow 0} \frac{1}{\Delta m} \left( \frac{\Delta v}{v_o} \right) \approx \frac{f_o}{\rho_l v_l} \left( \frac{d \log_e v}{dz} \right)_{z_0}$$

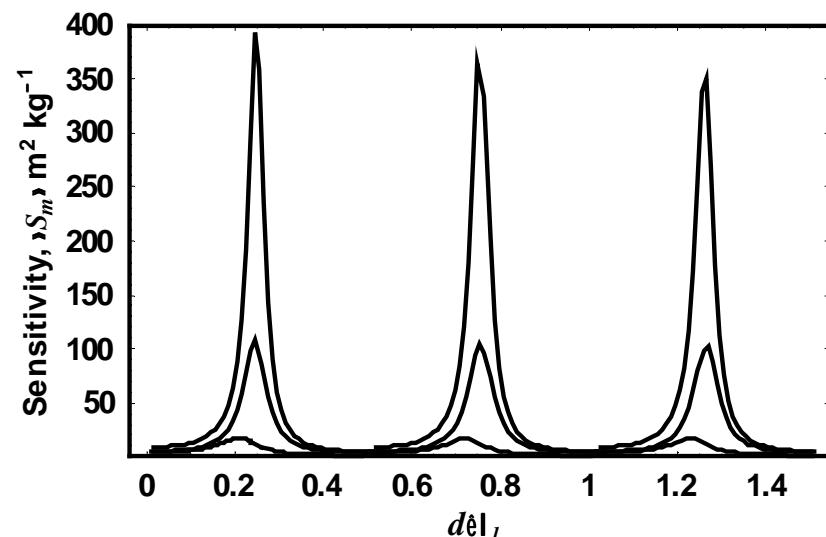
$\Delta m$  is mass per unit area being sensed,  $z=df/v_l$  is the normalized thickness

"Rigid" mass  $\Rightarrow$  Mass sensitivity is slope of dispersion curve<sup>1</sup>

## Love Waves



## Layer-Guided SH-APMs



<sup>1</sup>McHale *et al*, J. Appl. Phys. (2002) 91, 9701-9710.

# Dispersion and Group Velocity

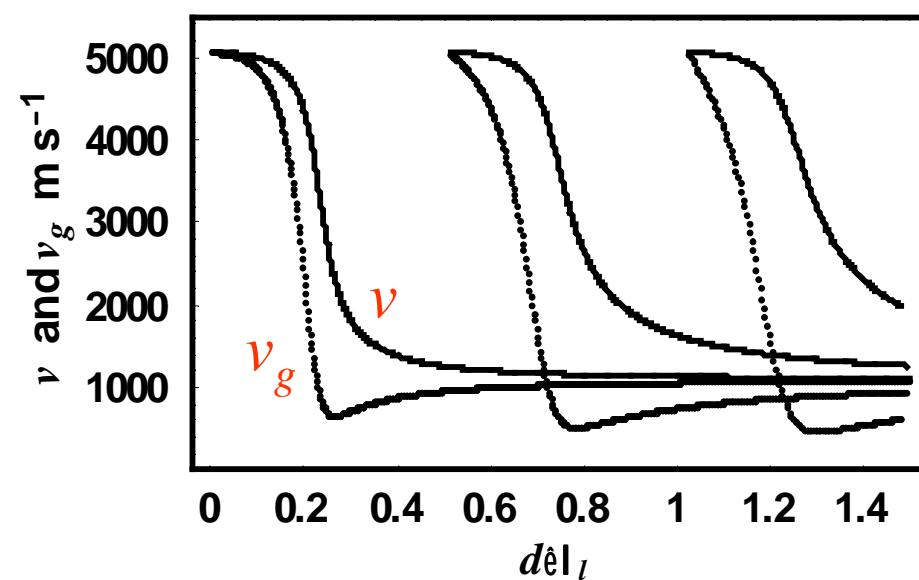
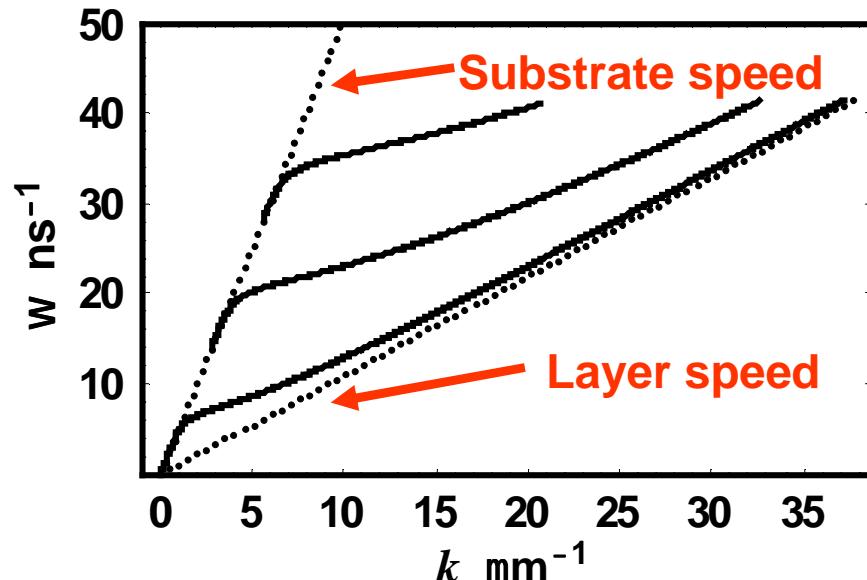
- Guiding Layer Induces Dispersion<sup>1,2</sup>

Phase velocity  
Group velocity

$$v = f\lambda \quad \text{or} \quad v = \omega/k$$
$$v_g = d\omega/dk$$

Group velocity is slope of the  $(\omega, k)$  dispersion curve

Example 0.25  $\mu\text{m}$  polymer guiding layer on Quartz with  $w \rightarrow \infty$



<sup>1</sup>McHale *et al*, "Sensitivity from group and phase .... ", J. Appl.Phys. (2002) vol 92

<sup>2</sup>Martin *et al*, "Experimental study of Love wave .... ", IEEE Sensor Journal (2002)

# Group Velocity Mass Sensitivity

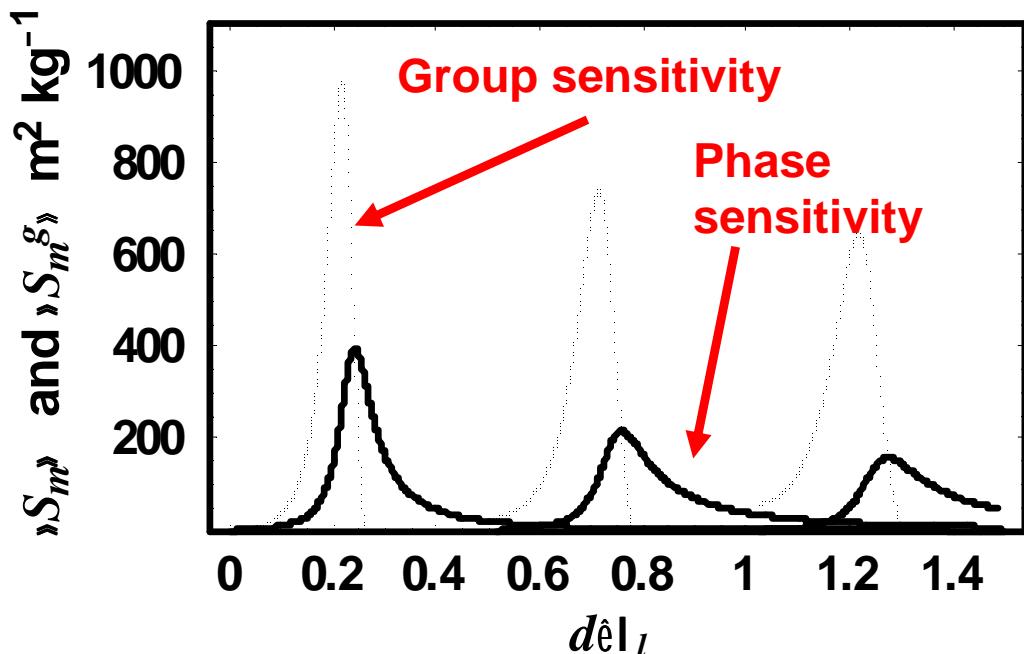
"Rigid" mass

$$S_m \approx \frac{1}{\rho_l d} \left( 1 - \frac{v}{v_g} \right) = \frac{1}{\rho_l d} \frac{(v_g - v)}{v_g}$$

Mass sensitivity is fractional deviation of the phase velocity from the group velocity divided by mass per unit area due to the guiding layer

- Define a Group Velocity Sensitivity

$$S_m^g = \frac{f_o}{\rho_l v_l} \left( \frac{d \log_e v_g}{dz} \right)_{z=z_o}$$



# Polymer Waveguide with Polymer Loading

## Generalized Sauerbrey Equation

Complex velocity shift

$$\frac{\Delta v}{v_o} \approx \left( \frac{1 - v_f^2/v_o^2}{1 - v_l^2/v_o^2} \right) \left( \frac{d \log_e v}{dz} \right)_{z=z_0} \left( \frac{\tan(T_f^o h)}{T_f^o h} \right) \frac{\omega \rho_f h}{2\pi v_l^\infty \rho_l}$$

Complex slope factor  
from polymer waveguide

$\tan x/x$  factor gives mass/liquid loading limits

### Care Needed

1. Dispersion
2. Slope depends on  $\omega$

$$\left( \frac{\tan(T_f^o h)}{T_f^o h} \right) \rightarrow \begin{cases} 1 & h \rightarrow 0 \\ \frac{-\sqrt{-2j}}{2h(1 - v_f^2/v_o^2)} \sqrt{\frac{2\eta_f}{\omega \rho_f}} & h \rightarrow \infty \text{ and } \omega \tau \rightarrow 0 \end{cases}$$

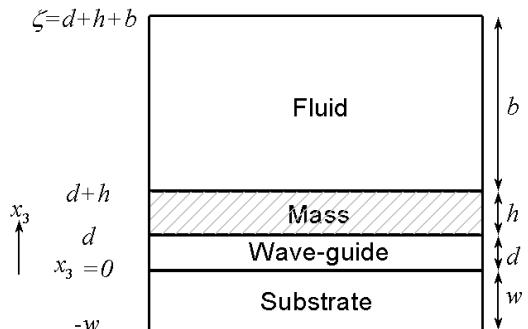
solid limit    liquid limit

<sup>1</sup>McHale et al, "Mass, liquid and polymer sensitivity .....", Accepted J. Appl. Phys. (2002).

# Insertion Loss for Polymer Waveguide

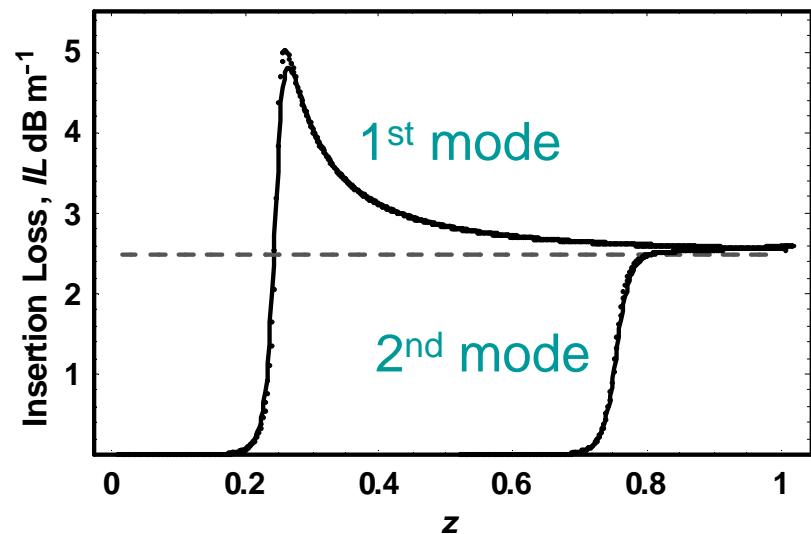
## Cases considered

1. Wave-guide layer is viscoelastic
2. Mass layer deposited from liquid or from vacuum
3. Mass may be omitted (i.e. liquid phase sensitivity)

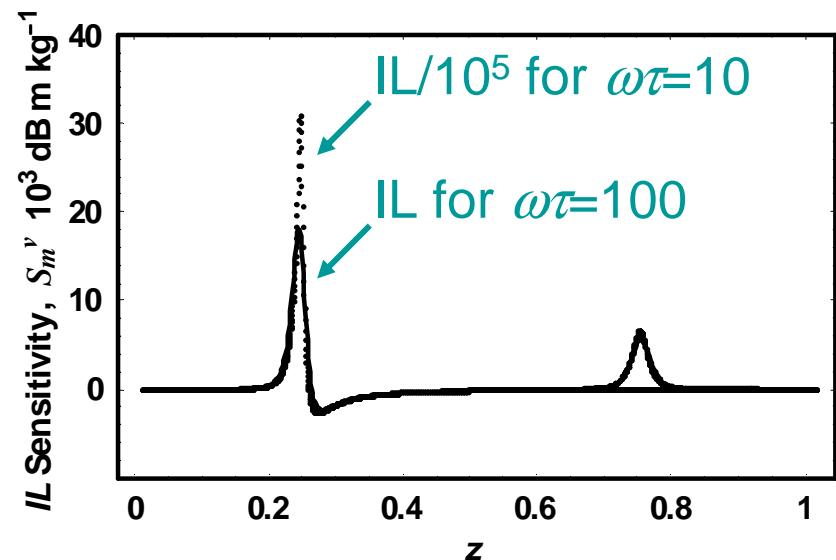


Mass/liquid sensitivity can be derived for phase velocity & insertion loss

## Love Wave Insertion Loss



## Love Wave IL Sensitivity



<sup>1</sup>McHale *et al*, "Mass, liquid and polymer sensitivity ....", Accepted J. Appl. Phys. (2002).

# Data for Layer-Guided SH-APM

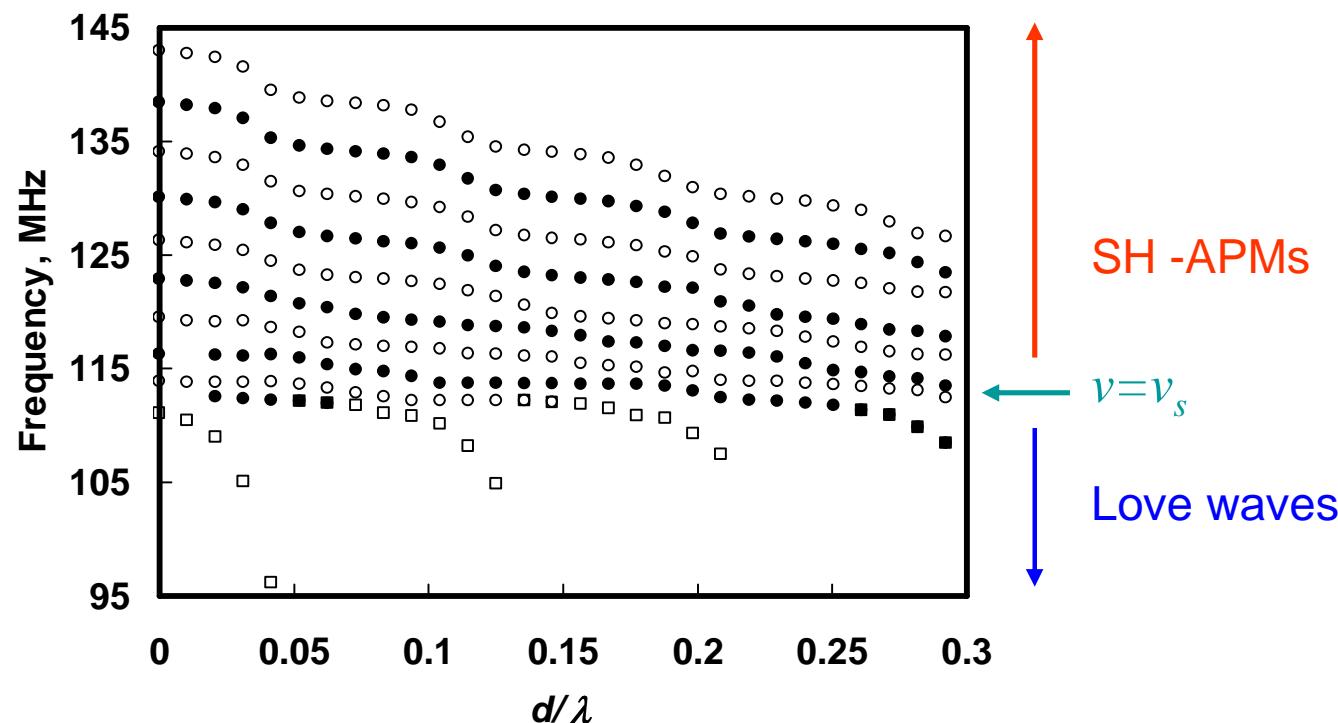
- Layer-Guided SH-APM Modes

Prop. Orthog. to x-axis of thinned ( $200\text{ }\mu\text{m}$ ) ST-Q substrate

110 MHz surface skimming bulk wave (SSBW)

SSBW  $\rightarrow$  Love wave by a spin-coated photoresist layer

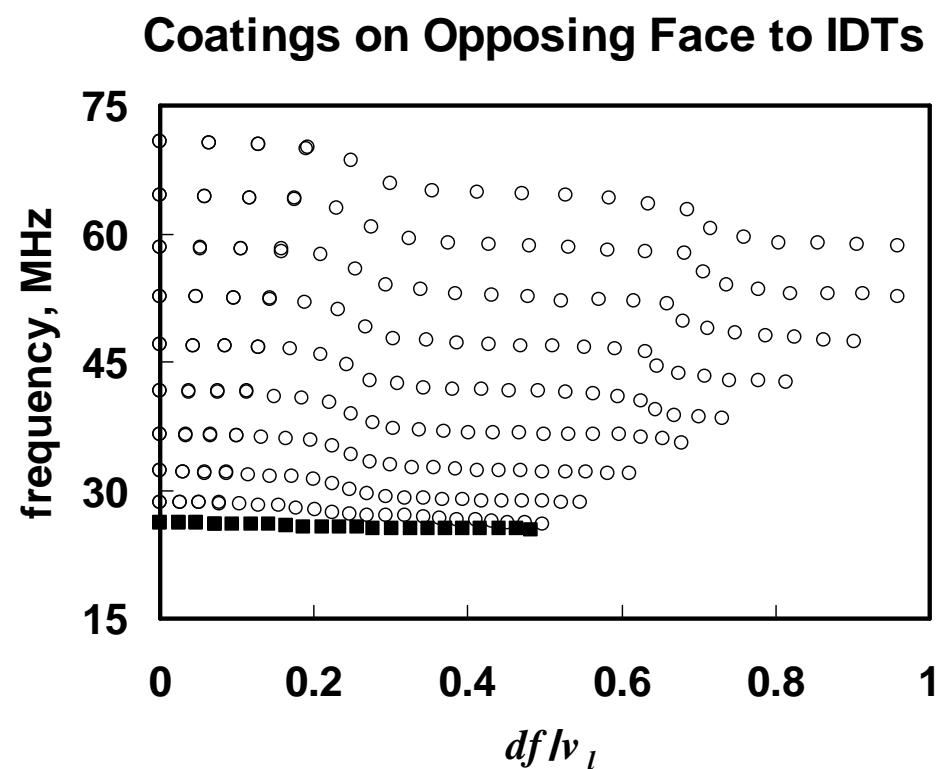
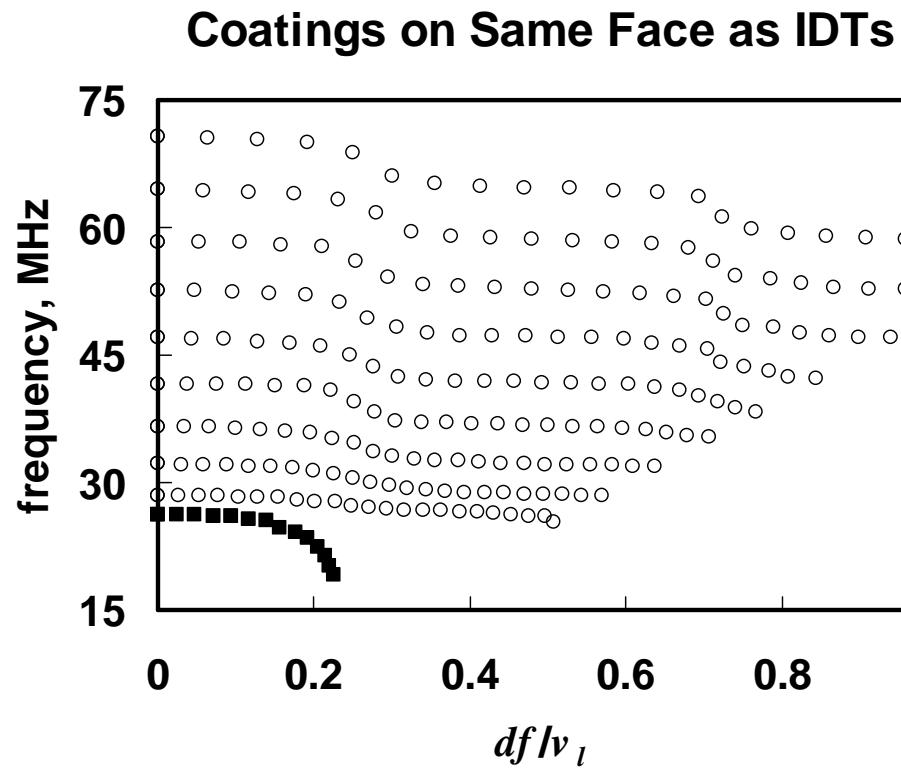
Old mask so insertion loss high (axis is  $d/\lambda$  with  $\lambda=\text{IDT period}$ )



# Experimental Data for SH-APM

- Layer-Guided SH-APM Modes<sup>1</sup>

Prop. Orthog. to x-axis of thinned (200  $\mu\text{m}$ ) ST-Q substrate  
25 MHz surface skimming bulk wave (SSBW)  
Plate modes clearly resolved



<sup>1</sup>F. Martin, PhD Thesis, Nottingham Trent University (2002)

# Group Velocity Data - Solids

- Phase and Group Velocity

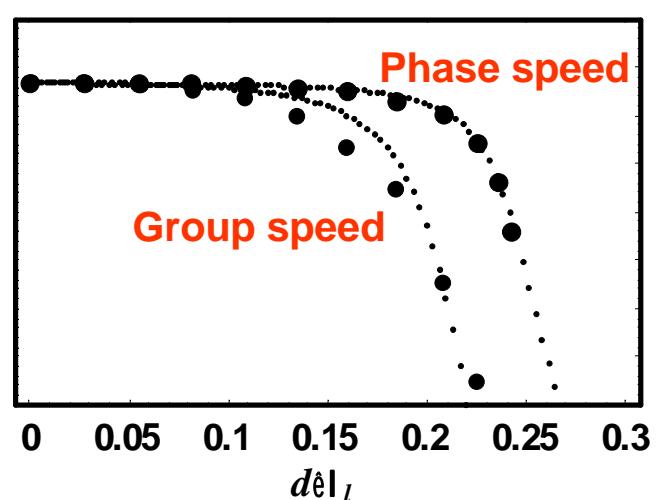
36° XY LiTaO<sub>3</sub>

Hardbaked photoresist

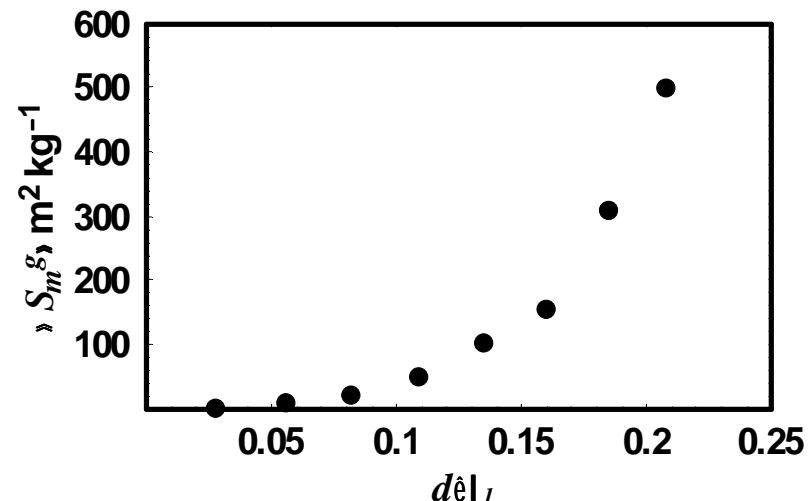
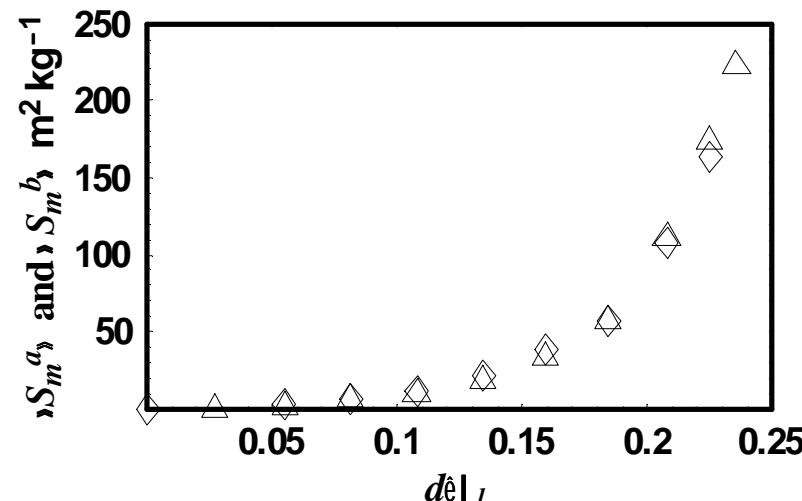
Dotted curves are fits

- Mass Sensitivity

$$S_m^b = \frac{1}{\rho_l d} \left( 1 - \frac{v}{v_g} \right) \quad S_m^a = \frac{1}{\rho_l} \left( \frac{d \log_e v}{dx} \right)_{x=d}$$



$$S_m^g = \frac{1}{\rho_l} \left( \frac{d \log_e v_g}{dx} \right)_{x=d}$$



# Group Velocity Data - Liquids

- **Deposition of Guiding Layer**

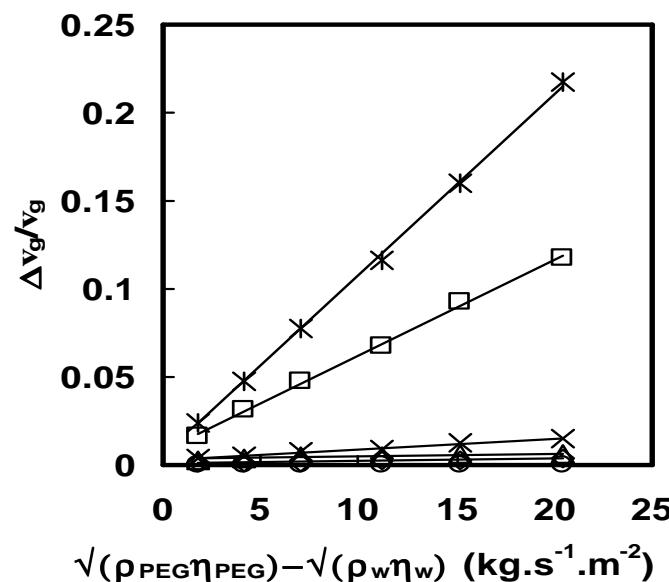
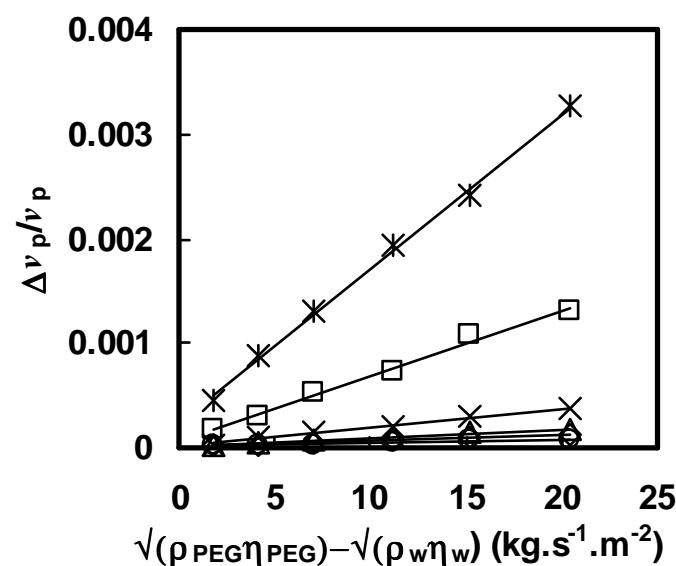
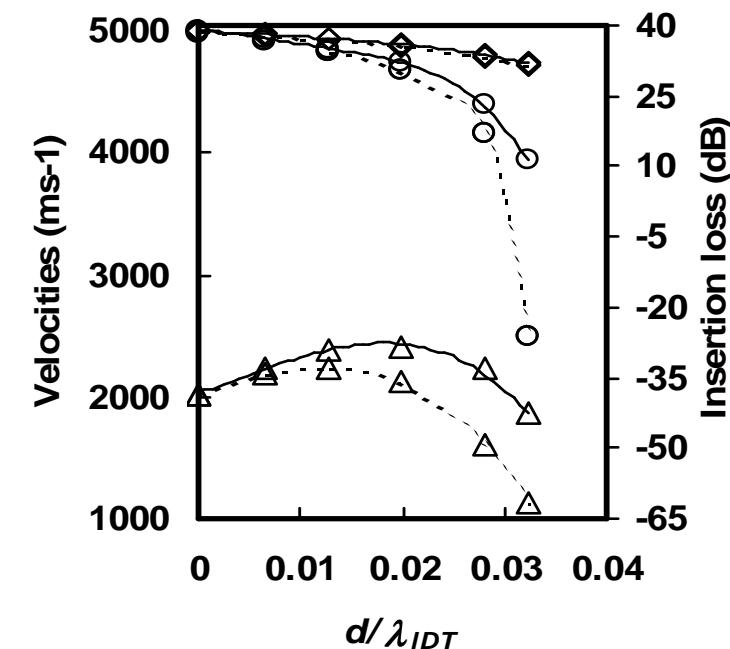
ST-Quartz + photoresist

Poly (ethylene glycol) solutions

◊ = Phase velocity      ○ = Grp Velocity

△ = Insertion loss      Dotted = water

- **Sensitivity at Operating Points**  
Phase and group velocities



# Love Waves and Higher Frequency

- Established QCM Sensor Principle

$$\begin{array}{lll} \text{Mass sensitivity} & \propto & \text{Fundamental frequency} \\ \text{Higher frequency} & \Rightarrow & \text{Higher mass sensitivity} \end{array}$$

- Love Waves on a (Semi-) Infinite Substrate

Controlling dimensionless variable is  $z = d/\lambda_l = df/v_l$

$$S_m = \lim_{\Delta m \rightarrow 0} \frac{1}{\Delta m} \left( \frac{\Delta v}{v_o} \right) \approx \frac{f_o}{\rho_l v_l} \left( \frac{d \log_e v}{dz} \right)_{z_0}$$

$$\begin{array}{lll} \text{Mass Sensitivity} & \propto & \text{Frequency} \times \text{Function of } z_0 \\ \text{Normalized thickness at operating point} & & z_0 \propto d \times f \end{array}$$

# Higher Frequency Operation<sup>1,2</sup>

## Routes

1. Increase fundamental frequency
2. Hop the frequency to a harmonic

## Issues

1. Change of Love wave mode?
2. Const. guiding layer thickness?

- Frequency Increase at Constant  $z_0$

Reduce  $d$  as  $1/f$

$\Rightarrow$  No change on dispersion curve

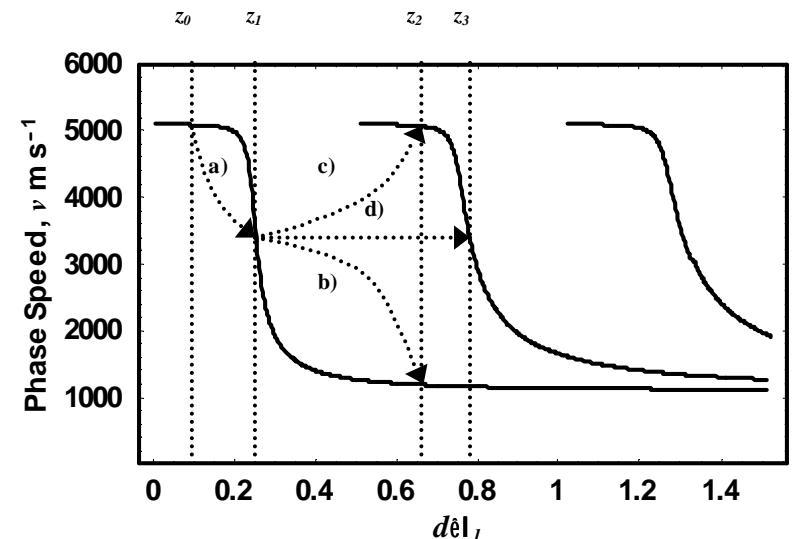
$\Rightarrow$  Mass sensitivity scales with  $f$

- Frequency Hopping at Constant  $d$

### Four example transitions

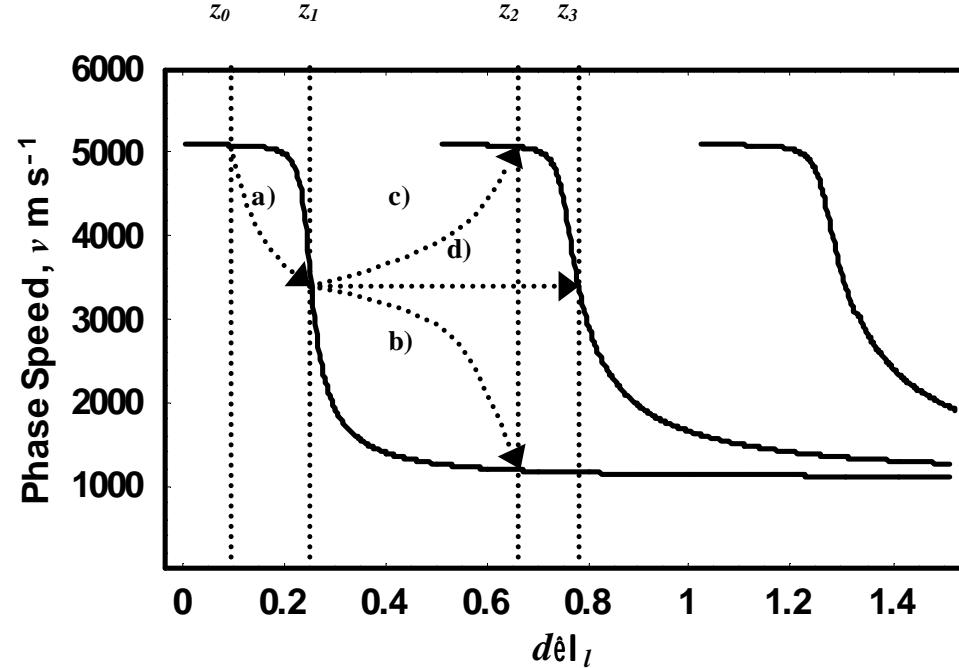
Same mode  $\Rightarrow$  lower/higher sensitivity

Change mode  $\Rightarrow$  lower/higher sensitivity



<sup>1</sup>M.I. Newton *et al*, Electron. Lett. (2001) 36, 340-341; <sup>2</sup>G. McHale *et al*, J. Appl. Phys. (2002), 91, 5735-5744

# Frequency Hopping Transitions



No Mode Change

- Transition a)  $\Rightarrow$  Higher mass sensitivity  
Transition b)  $\Rightarrow$  Lower mass sensitivity

Mode Change

- Transition c)  $\Rightarrow$  Lower mass sensitivity  
Transition d)  $\Rightarrow$  Higher mass sensitivity

Maximum Increase in Mass Sensitivity

Ratio of frequencies  $\times$  ratio of max slopes of modes  
i.e. scales by less than by the frequency ratio

# Summary

## Achievements

- Unifying theory  
Love wave and SH-APM's
- New sensor  
Layer-guided SH-APM's
- Mass/liquid sensitivity predictions  
Phase velocity and insertion loss  
Relation to group velocity
- Love wave frequency response  
Mode and non-mode changes

## Lessons

- Higher order Love waves  
from SH-APM's
- Guiding layer on SH-APM's  
significant increase in sensitivity
- Higher frequency  
Higher or lower sensitivity  
Frequency scaling of mode peak
- Love waves  $\Rightarrow$  strong dispersion  
Group and phase velocity differ

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The End

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